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Convergence Analysis of Parallel Iterative Algorithm for a New System of Nonconvex Variational Inequalities*

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Abstract: For a system of general nonconvex variational inequalities defined on uniformly prox-regular sets, we propose a parallel projection algorithm which converges to its solution and common fixed points of two Lipschitzian mappings. We further consider the convergence of the algorithm under some suitable conditions. Results presented in this article improve and extend the previously known results for the variational inequalities and related optimization problems.

Key words: system of general nonconvex variational inequalities; uniform prox-regular set; relaxed cocoercive mapping; strongly monotone operator; Lipschitzian mapping

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1 Introduction

Variational inequality theory was introduced by Stampacchia[1] in the early 1960s, which is an important branch of applicable mathematical and with a widerange of applications in nonlinear optimization theory, differential equation, control problem, equilibrium theory. One of the basic problem in variational inequalities is the existence of solutions problem and the research of the iterative method for its solutions. There are a lot of methods can used to solving variational inequalities, such as projection method and its variant forms, Wiener-Hopf equations, auxiliary principle and so on, see [1-13] and the references therein.

In recent years, researchers had a keen interest in uniformly prox-regular sets and the nonconvex variational inequalities problems NCVIP, which is defined uniformly prox-regular sets[2-5]. As known to all that uniformly prox-regular sets are nonconvex sets and include convex sets as special case. Noor[4] first introduced and research a class of NCVIP. Moreover, he proved the equivalence between the NCVIP and the fixed-point problems by using the projection technique. This equivalent formulation is often used to discuss the existence and algorithm of the solution of the NCVIP. Noor[5] also proposed some other methods for solving a general NCVIP, such as projection methods and Wiener-Hopf equations technique. On the other hand, Verma[6] and Noor[7] proposed explicit projection methods for solving systems of variational inequalities and general variational inequalities on a closed convex subset of Hilbert space, respectively. Recently, many researchers began to consider variational inequalities system problems; see [8-10] and the references therein. In 2012, Wen et al.[10] generalized the nonconvex variational inequalities to a new system nonconvex variational inequalities and discussed the convergence of projection methods for the new system of general nonconvex variational inequali-

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ties.

In this article, we introduce and consider a new and more general system of general nonconvex, variational inequalities problems SGNCVIP. The GNCVIP includes the system of variational inequalities involving two different nonlinear operators, the general nonconvex variational inequalities and the systems of variational inequalities defined on closed convex sets as special cases. In this paper, we first show that projection technique can be extended to the new system of general nonconvex variational inequalities on uniformly prox-regular sets, and then propose a new parallel algorithm which converges to its solution. Unlike the algorithm 3.1 in [10], an important feature of the new parallel algorithm is that it has the suitability for implementing on multiprocessor computer. We also consider the convergence of the parallel projection algorithm under some suitable mild conditions. The results presented in this article improve and extend the previously known results for the variational inequalities and related optimization problems.

2 Preliminaries

In this section, we present some basic definitions and preliminary results that will be used throughout the paper. The Hilbert space is denoted by H and we use $\langle \cdot , \cdot \rangle$ and $\| \cdot \|$ denote the inner productand norm of H, respectively. The nonempty closed convex subset of H is denoted by K. The identity operator is denoted by I.

Definition 1[2-3] The proximal normal cone of K at $u \in H$ is given by

$$N_K^P(u) := \{\xi \in H : u \in P_k(u+\alpha\xi)\}, \tag{1}$$

where $\alpha > 0$ is a constant and $P_K(u)=\{u^* \in K : d_K(u)=\| u-u^* \|\}$, $d_K(u)=\inf_{v\in K}\| v-u \|$.

The proximal normal cone $N_K^P(u)$ has the following characterization.

Lemma 1[2-3] Let K be a nonempty closed convex subset in H. Then $\xi \in N_K^P(u)$ if and only if $\exists\ \alpha > 0$ such that

$$\langle \xi, v-u \rangle \leqslant \alpha \| v-u \|^2, \forall v \in K. \tag{2}$$

Definition 2[2-3] The Clarke normal cone, denoted by $N_K^C(u)$, is defined as

$$N_K^C(u)=\overline{co}(N_K^P(u)), \tag{3}$$

where $\overline{co}(A)$ is the closure of the convex hull of the set A.

Definition 3[2-3] For $r \in (0, \infty]$, $K_r \subset K$ is said to be normalized uniformly r-prox-regular if and only if $\forall u \in K_r$, $0 \neq \xi \in N_{K_r}^P(u)$, we have

$$\left\langle \frac{\xi}{\| \xi \|}, v-u \right\rangle \leqslant \frac{1}{2r} \| v-u \|^2, \forall v \in K_r. \tag{4}$$

Remark 1 From [2-3] we know, if $r=\infty$, then $K_r=K$.

Let K_r be a uniformly r-prox-regular (nonconvex) set, $T_1, T_2 : K_r \times K_r \to K_r$ and $g, h : H \to K_r$ be different nonlinear operators, respectively. For any given constants $\rho_1 > 0$ and $\rho_2 > 0$, finding x^*, $y^* \in K_r$, such that

$$\begin{cases} \langle \rho_1 T_1(y^*, x^*) + x^* - g_1(y^*), g_1(x) - x^* \rangle + \dfrac{1}{2r} \| g_1(x) - x^* \|^2 \geqslant 0, \forall x \in H : g_1(x) \in K_r \\ \langle \rho_2 T_2(x^*, y^*) + y^* - g_2(x^*), g_2(x) - y^* \rangle + \dfrac{1}{2r} \| g_2(x) - y^* \|^2 \geqslant 0, \forall x \in H : g_2(x) \in K_r \end{cases}, \tag{5}$$

which is called a new system of general nonconvex variational inequalities. Some special cases of (5) as follows:

1) If $g_1 = g_2 = I$, then (5) is deformed into the following forms: finding $x^*, y^* \in K_r$, such that

$$\begin{cases} \langle \rho_1 T_1(y^*, x^*) + x^* - y^*, x - x^* \rangle + \dfrac{1}{2r} \| g_1(x) - x^* \|^2 \geqslant 0, \forall x \in H : g_1(x) \in K_r, \rho_1 > 0 \\ \langle \rho_2 T_2(x^*, y^*) + y^* - x^*, x - y^* \rangle + \dfrac{1}{2r} \| g_2(x) - y^* \|^2 \geqslant 0, \forall x \in H : g_2(x) \in K_r, \rho_2 > 0 \end{cases}, \tag{6}$$

which appears to be a new one.

2) If $r=\infty$, then $K_r=K$, then (5) is equivalent to finding $x^*, y^* \in K$, such that

$$\begin{cases}\langle \rho_1 T_1(y^*, x^*)+x^*-g_1(y^*), g_1(x)-x^*\rangle \geqslant 0, \forall x\in H: g_1(x)\in K, \rho_1>0 \\ \langle \rho_2 T_2(x^*, y^*)+y^*-g_2(x^*), g_2(x)-y^*\rangle \geqslant 0, \forall x\in H: g_2(x)\in K, \rho_2>0\end{cases}, \tag{7}$$

which is known as the system of general variational inequalities involving four different nonlinear operators, introduced, and studied by Noor [11].

3) If $g_1=g_2=I, T_1, T_2: K\rightarrow K$, then (7) is deformed into the following forms: finding $x^*, y^*\in K$, such that

$$\begin{cases}\langle \rho_1 T_1(y^*)+x^*-y^*, x-x^*\rangle \geqslant 0, \forall x\in K, \rho_1>0 \\ \langle \rho_2 T_2(x^*)+y^*-x^*, x-y^*\rangle \geqslant 0, \forall x\in K, \rho_2>0\end{cases}, \tag{8}$$

which is known as the system of nonlinear variational inequalities involving two different nonlinear operators. If $T_1=T_2=T$, problem (8) reduces to the system of variational inequalities, which was introduced and studied by Verma [6].

4) If $T_1=T_2=T: K_r\rightarrow K_r$, and $x^*=y^*=u$, then (6) is equivalent to finding $u\in K_r$, such that

$$\langle Tu, v-u\rangle+\frac{1}{2r}\| v-u\|^2\geqslant 0, \forall v\in K_r, \tag{9}$$

5) If $r=\infty$, then problem (9) is equivalent to finding $u\in K_r$, such that

$$\langle Tu, v-u\rangle \geqslant 0, \forall v\in K_r, \tag{10}$$

which is the normal nonconvex variational inequality introduced and studied by Noor[4,20]. It is well known that problem (10) is equivalent to finding $u\in K_r$ such that

$$0\in Tu+N_{K_r}^P(u), \tag{11}$$

We now recall the well-known proposition which summarizes some important properties of the uniform prox-regular sets.

Lemma 2[2-4,10] Let K be a nonempty closed subset of H, $r\in(0,\infty]$ and set $K_r=\{u\in H: d(u,K)<r\}$. If K_r is uniformly prox-regular, then: i) $\forall u\in K_r$, $P_{K_r}(u)\neq\varnothing$; ii) $\forall r'\in(0,r), P_{K_r}$ is δ-Lipchitz continuous, where $\delta=\frac{r}{r-r'}$; iii) $N_{K_r}^P(u)$ is closed.

Definition 4 An operator $g: H\rightarrow H$ is said to be

1) ξ-strongly monotone if and only if $\forall\ x, x'\in H$, $\exists\ \xi>0$ such that

$$\langle g(x)-g(x'), x-x'\rangle \geqslant \xi\| x-x'\|^2; \tag{12}$$

2) η-Lipchitz continuous if and only if $\forall\ x, x'\in H$, $\exists\ \eta>0$ such that

$$\| g(x)-g(x')\| \leqslant \eta\| x-x'\|. \tag{13}$$

An operator $T: H\times H\rightarrow H$ is said to be

3) relaxed (ω, t)-cocoercive with respect to the first variable if and only of $\forall\ x, x'\in H$, $\exists\ t>0$ and $\omega>0$ such that

$$\langle T(x,\cdot)-T(x',\cdot), x-x'\rangle \geqslant -\omega\| T(x,\cdot)-T(x',\cdot)\|^2+t\| x-x'\|^2; \tag{14}$$

4) μ-Lipschitz continuous with respect to the first variable if and only if $\forall\ x, x'\in H$, $\exists\ \mu>0$ such that

$$\| T(x,\cdot)-T(x',\cdot)\| \leqslant \mu\| x-x'\|; \tag{15}$$

5) γ-Lipschitz continuous with respect to the second variable if and only if $\forall\ y, y'\in H$, $\exists\ \gamma>0$ such that

$$\| T(\cdot, y)-T(\cdot, y')\| \leqslant \gamma\| x-x'\|. \tag{16}$$

Remark 2 From the above definition, the identity operator I be a 1-strongly monotone and 1-Lipschitz continuous mapping. If operator $g: H\rightarrow H$ is ξ-strongly monotone and η-Lipchitz continuous, then $\eta\geqslant\xi$. If $T: H\times H\rightarrow H$ be a strongly monotone mapping with respect to the first or second variable, then T must be a relaxed cocoercive mapping with respect to the first or second variable.

Lemma 3[10] $x^*, y^*\in K_r$ is a solution of the system of general nonconvex variational inequalities problem (1), if and only if

$$\begin{cases} x^* = P_{K_r}[g_1(y^*) - \rho_1 T_1(y^*, x^*)] \\ y^* = P_{K_r}[g_2(x^*) - \rho_2 T_2(x^*, y^*)] \end{cases}, \tag{17}$$

where P_{K_r} is the projection of H onto the uniformly prox-regular set K_r.

Lemma 4[13] Let $\{a_n\} \subset \mathbf{R}^+$, $\{b_n\} \subset \mathbf{R}^+$. We further assumed that

$$a_{n+1} \leqslant (1-d_n)a_n + b_n,\ \forall n \geqslant n_0, \tag{18}$$

where n_0 is some nonnegative integer, $d_n \in (0,1)$ with $\sum_{n=0}^{\infty} d_n = \infty$ and $b_n = o(d_n)$, then $a_n \to 0$ as $n \to \infty$.

3 Main results

In this section we use Lemma 3 propose a relaxed two-step algorithm for solving problem (5), furthermore we consider the convergence of this algorithm.

Algorithm 1 For any initial initial points $x_0, y_0 \in K_r$, the sequences $\{x_n\}$, $\{y_n\}$ are generated by the following iterative manner:

$$\begin{cases} x_{n+1} = (1-\alpha_n)x_n + \alpha_n S_1(P_{K_r}(g_1(y_n) - \rho_1 T_1(y_n, x_n))) \\ y_{n+1} = (1-\beta_n)y_n + \beta_n S_2(P_{K_r}(g_2(x_n) - \rho_2 T_2(x_n, y_n))) \end{cases}, \tag{19}$$

where K_r is a uniformly prox-regular set, $\{\alpha_n\}, \{\beta_n\}$ are two sequences in $[0,1]$ and ρ_1, ρ_2 is positive real numbers, S_1, S_2 are two Lipschitzian mappings.

Let $F(S_i) = \{x \in H : S_i x = x\}$, $F(S) = \bigcap_{i=1}^{2} F(S_i)$, the solutions set of (5) is denoted by $SOL(5)$. We first prove the following Lemma, which will be helpful to prove our main result of in this section.

Lemma 5 Let H be a real Hilbert space. Let $\{x_n\}$ and $\{y_n\}$ be sequences in H such that

$$\| x_{n+1} - x^* \| + \| y_{n+1} - y^* \| \leqslant \max\{(1-r_n)(1-s_n)\}(\| x_n - x^* \| + \| y_n - y^* \|) \tag{20}$$

for some $x^*, y^* \in H$, where $\{r_n\}$ and $\{s_n\}$ are sequences in $(0,1)$ such that $\sum_{n=0}^{\infty} r_n = \infty$ and $\sum_{n=0}^{\infty} s_n = \infty$. Then $\{x_n\}$ and $\{y_n\}$ converges to x^* and y^*, respectively.

Proof Firstly, we define the norm $\| \cdot \|_1$ on $H \times H$ in the following form

$$\| (x, y) \|_1 = \| x \| + \| y \|,\ \forall (x, y) \in H \times H.$$

Then $(H \times H, \| \cdot \|_1)$ is a Banach space. Hence by the define of $\| \cdot \|_1$, (20) implies that

$$\| (x_{n+1}, y_{n+1}) - (x^*, y^*) \|_1 \leqslant \max\{(1-r_n)(1-s_n)\} \| (x_n, y_n) - (x^*, y^*) \|_1.$$

Using Lemma 4, we have

$$\lim_{n \to \infty} \| (x_n, y_n) - (x^*, y^*) \|_1 = \lim_{n \to \infty} (\| x_n - x^* \| + \| y_n - y^* \|) = 0.$$

Therefore, $\{x_n\}$ and $\{y_n\}$ converges to x^* and y^*, respectively. This completes the proof.

We now present the approximation solvability of the problem (5).

Theorem 1 Let H be a real Hilbert space and $K \subset H$ be a nonempty closed convex set, $K_r \subset K$ be a closed uniformly prox-regular set. Let P_{K_r} be a Lipschitz continuous operator with constant $\delta = \frac{r}{r - r'}$. Let $T_i : K_r \times K_r \to K_r$ and $g_i : K_r \to K_r$ be mappings such that T_i is relaxed (ω_i, t_i)-cocoercive, μ_i-Lipschitz continuous with respect to the first variable, γ_i-Lipschitz continuous with respect to the second variable and g_i is η_i-Lipschitz continuous, ξ_i-strongly monotone mapping for $i = 1, 2$. Let $S_i : H \to H$ be ϑ_i-Lipschitzian mapping for $i = 1, 2$ with $F(S) \neq \varnothing$, $\{\alpha_n\}$, $\{\beta_n\}$ are two sequences in $[0,1]$. Assume that the following assumptions hold:

i) $0 < \Omega_{1n} = \alpha_n(1 - \vartheta\delta\rho_1\gamma_1) - \beta_n\vartheta\delta(\psi_2 + \theta_2) < 1$,

ii) $0 < \Omega_{2n} = \beta_n(1 - \vartheta\delta\rho_2\gamma_2) - \alpha_n\vartheta\delta(\psi_1 + \theta_1) < 1$,

iii) $\sum_{n=0}^{\infty} \Omega_{1n} = \infty$, and $\sum_{n=0}^{\infty} \Omega_{2n} = \infty$,

where $\vartheta = \max\{\vartheta_1, \vartheta_2\}$ and $\psi_i = \sqrt{1 - 2\xi_i + \eta_i^2}$, $\theta_i = \sqrt{1 + 2\rho_i\omega_i\mu_i^2 - 2\rho_i t_i + \rho_i^2\mu_i^2}$, $i = 1, 2$. If $SOL(5) \cap F(S) \neq \varnothing$, then the sequences $\{x_n\}$and $\{y_n\}$ generated by the Algorithm 1 converges to x^* and y^*, respectively, such that $(x^*, y^*)\in SOL(5)$ and $\{x^*, y^*\}\in F(S)$.

Proof Let us have $(x^*, y^*)\in SOL(5)$ and $\{x^*, y^*\}\in F(S)$. By Lemma 3, we have

$$\begin{cases} x^*=P_{K_r}[g_1(y^*)-\rho_1 T_1(y^*, x^*)] \\ y^*=P_{K_r}[g_2(x^*)-\rho_2 T_2(x^*, y^*)] \end{cases},$$

Also since $\{x^*, y^*\}\in F(S)$, we have

$$\begin{cases} x^*=S_1(P_{K_r}(g_1(y^*)-\rho_1 T_1(y^*, x^*))) \\ y^*=S_2(P_{K_r}(g_2(x^*)-\rho_2 T_2(x^*, y^*))) \end{cases}, \tag{21}$$

To prove the result, we first evaluate $\| x_{n+1}-x^* \|$ for all $n\geqslant 0$. Using (19) and (21), we obtain

$$\begin{aligned} &\| x_{n+1}-x^* \| = \| (1-\alpha_n)x_n+\alpha_n S_1(P_{K_r}(g_1(y_n)-\rho_1 T_1(y_n, x_n)))-x^* \| \leqslant \\ &(1-\alpha_n)\| x_n-x^* \| +\alpha_n \| S_1(P_{K_r}(g_1(y_n)-\rho_1 T_1(y_n, x_n)))-S_1(P_{K_r}(g_1(y^*)-\rho_1 T_1(y^*, x^*)))\| \leqslant \\ &(1-\alpha_n)\| x_n-x^* \| +\alpha_n\vartheta_1\delta \| g_1(y_n)-g_1(y^*)-\rho_1(T_1(y_n, x_n)-T_1(y^*, x^*))\| \leqslant \\ &(1-\alpha_n)\| x_n-x^* \| +\alpha_n\vartheta_1\delta \| g_1(y_n)-g_1(y^*)-(y_n-y^*)\| + \\ &\alpha_n\vartheta_1\delta \| y_n-y^*-\rho_1(T_1(y_n, x_n)-T_1(y^*, x_n))\| +\alpha_n\vartheta_1\delta\rho_1 \| T_1(y^*, x_n)-T_1(y^*, x^*)\| . \end{aligned} \tag{22}$$

Since g_1 is η_1-Lipschitz continuous and ξ_1-strongly monotone, we have

$$\begin{aligned} &\| g_1(y_n)-g_1(y^*)-(y_n-y^*)\|^2 = \| g_1(y_n)-g_1(y^*)\|^2-2\langle g_1(y_n)-g_1(y^*), y_n-y^*\rangle+\| y_n-y^* \|^2\leqslant \\ &\eta_1^2 \| y_n-y^* \|^2-2\xi_1 \| y_n-y^* \|^2+\| y_n-y^* \|^2=(1-2\xi_1+\eta_1^2)\| y_n-y^* \|^2. \end{aligned} \tag{23}$$

By the assumption about T_1 and the definition of relaxed cocoercive and Lipschitz continuous, we have

$$\begin{aligned} &\| y_n-y^*-\rho_1(T_1(y_n, x_n)-T_1(y^*, x_n))\|^2 = \| y_n-y^* \|^2-2\rho_1\langle T_1(y_n, x_n)-T_1(y^*, x_n), y_n-y^*\rangle+ \\ &\rho_1^2 \| T_1(y_n, x_n)-T_1(y^*, x_n)\|^2\leqslant \| y_n-y^* \|^2+2\rho_1\omega_1 \| T_1(y_n, x_n)-T_1(y^*, x_n)\|^2-2\rho_1 t_1 \| y_n-y^* \|^2+ \\ &\rho_1^2 \| T_1(y_n, x_n)-T_1(y^*, x_n)\|^2\leqslant \| y_n-y^* \|^2+2\rho_1\omega_1\mu_1^2 \| y_n-y^* \|^2-2\rho_1 t_1 \| y_n-y^* \|^2+ \\ &\rho_1^2\mu_1^2 \| y_n-y^* \|^2=(1+2\rho_1\omega_1\mu_1^2-2\rho_1 t_1+\rho_1^2\mu_1^2)\| y_n-y^* \|^2. \end{aligned} \tag{24}$$

By γ_1-Lipschitz continuity of T_1 with respect to second variable, we have

$$\| T_1(y^*, x_n)-T_1(y^*, x^*))\leqslant\gamma_1 \| x_n-x^* \| . \tag{25}$$

Substituting (23)~(25) into (22), we have

$$\| x_{n+1}-x^* \| \leqslant(1-\alpha_n+\alpha_n\vartheta_1\delta\rho_1\gamma_1)\| x_n-x^* \| +\alpha_n\vartheta_1\delta(\psi_1+\theta_1)\| y_n-y^* \| , \tag{26}$$

where $\psi_1=\sqrt{1-2\xi_1+\eta_1^2}$, $\theta_1=\sqrt{1+2\rho_1\omega_1\mu_1^2-2\rho_1 t_1+\rho_1^2\mu_1^2}$.

Similarly, we have

$$\| y_{n+1}-y^* \| \leqslant\beta_n\vartheta_2\delta(\psi_2+\theta_2)\| x_n-x^* \| +(1-\beta_n+\beta_n\vartheta_2\delta\rho_2\gamma_2)\| y_n-y^* \| , \tag{27}$$

where $\psi_2=\sqrt{1-2\xi_2+\eta_2^2}$, $\theta_2=\sqrt{1+2\rho_2\omega_2\mu_2^2-2\rho_2 t_2+\rho_2^2\mu_2^2}$.

Adding (26) and (27), taking $\vartheta=\max\{\vartheta_1, \vartheta_2\}$ we get

$$\begin{aligned} &\| x_{n+1}-x^* \| +\| y_{n+1}-y^* \| \leqslant(1-\alpha_n+\alpha_n\vartheta\delta\rho_1\gamma_1)\| x_n-x^* \| +\alpha_n\vartheta\delta(\psi_1+\theta_1)\| y_n-y^* \| + \\ &\beta_n\vartheta\delta(\psi_2+\theta_2)\| x_n-x^* \| +(1-\beta_n+\beta_n\vartheta\delta\rho_2\gamma_2)\| y_n-y^* \| = \\ &[1-(\alpha_n(1-\vartheta\delta\rho_1\gamma_1)-\beta_n\vartheta\delta(\psi_2+\theta_2))\| x_n-x^* \| +[1-(\beta_n(1-\vartheta\delta\rho_2\gamma_2)-\alpha_n\vartheta\delta(\psi_1+\theta_1))\| y_n-y^* \| \leqslant \\ &\max\{(1-\Omega_{1n}, (1-\Omega_{2n})\}(\| x_n-x^* \| +\| y_n-y^* \|), \end{aligned}$$

where $\Omega_{1n}=\alpha_n(1-\vartheta\delta\rho_1\gamma_1)-\beta_n\vartheta\delta(\psi_2+\theta_2)$, $\Omega_{2n}=\beta_n(1-\vartheta\delta\rho_2\gamma_2)-\alpha_n\vartheta\delta(\psi_1+\theta_1)$. By the assumptions and Lemma 5, we get that the sequences $\{x_n\}$ and $\{y_n\}$ converges to x^* and y^*, respectively. This completes the proof.

Remark 2 Theorem 1 improve and extend the Theorem 4.1 in Wen et al.[10], which itself is an extension and improvement of the main result in Noor[11] and Verma[6].

4 Conclusion

A new system of general nonconvex variational inequalities defined on uniformly prox-regular sets are considered in this paper. We propose a parallel projection algorithm which converges to its solution and common fixed points of two Lipschitzian mappings. We also prove that the algorithm is convergent. The results of this paper extend some corresponding results on the variational inequalities and related optimization problems. It is an interesting open problem to implement these algorithms for solving the system of variational inequalities numerically and compare its efficiency with other iterative methods.

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运筹学与控制论

一类新的推广非凸变分不等式的平行投影算法

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摘要：对定义在一致临近正则集上的一类新的推广的非凸变分不等式，本文提出了一个平行投影算法，算法的收敛点既是该变分不等式的解，又是两个 Lipschitz 映像的不动点。进一步，本文在适当条件下证明了该算法的收敛性。本文所得结论改进并推广了有关变分不等式和相关最优化问题的一些结果。

关键词：推广的非凸变分不等式；一致临近正则集；松弛强制映像；强单调算子；Lipschitz 连续映像

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